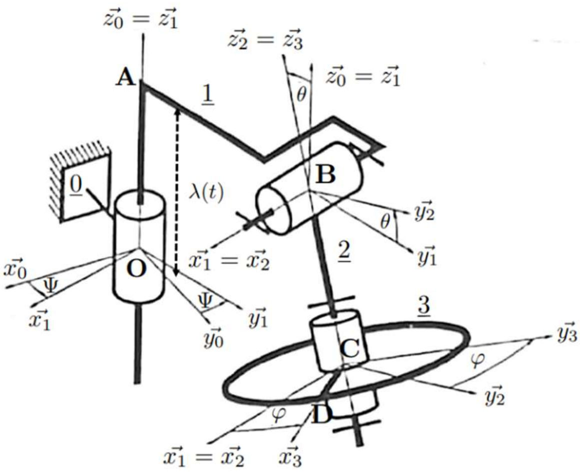
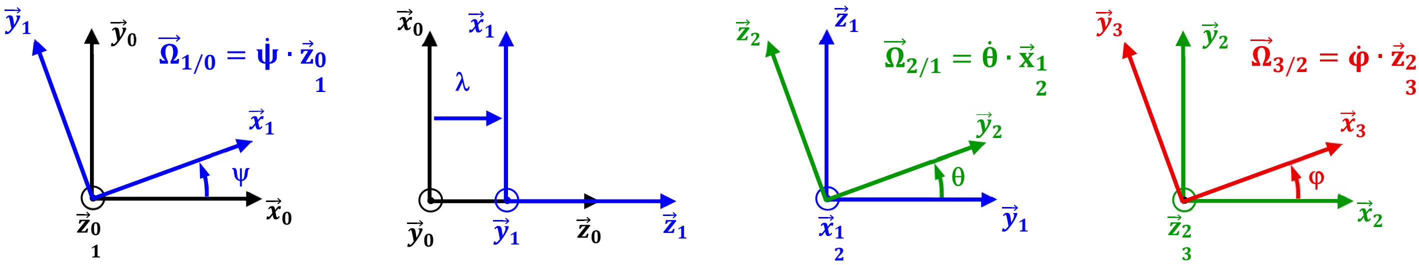
Manège TECHNO POWER
CORRIGÉ



Figures géométrales :



$$\vec{V}_{D,3/0} = \left.\frac{d\overrightarrow{OD}}{dt}\right|_0 = \left.\frac{d\left(\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right)}{dt}\right|_0 = \left.\frac{d(\lambda.\vec{z}_1)}{dt}\right|_0 + \left.\frac{d(a.\vec{y}_1)}{dt}\right|_0 + \left.\frac{d(-b.\vec{z}_2)}{dt}\right|_0 + \left.\frac{d(c.\vec{x}_3)}{dt}\right|_0$$

$$= \left.\frac{d(\lambda.\vec{z}_1)}{dt}\right|_0 + a \cdot \left.\frac{d\vec{y}_1}{dt}\right|_0 - b \cdot \left.\frac{d\vec{z}_2}{dt}\right|_0 + c \cdot \left.\frac{d\vec{x}_3}{dt}\right|_0$$

avec :

$$\left.\frac{d(\lambda.\vec{z}_1)}{dt}\right|_0 = \dot{\lambda} \cdot \vec{z}_1 + \lambda \cdot \left.\frac{d\vec{z}_1}{dt}\right|_0 = \dot{\lambda} \cdot \vec{z}_1 + \lambda \cdot \left(\left.\frac{d\vec{z}_1}{dt}\right|_1 + \vec{\Omega}_{1/0} \wedge \vec{z}_1\right) = \dot{\lambda} \cdot \vec{z}_1$$

$$\left.\frac{d\vec{y}_1}{dt}\right|_0 = \left.\frac{d\vec{y}_1}{dt}\right|_1 + \vec{\Omega}_{1/0} \wedge \vec{y}_1 = \dot{\psi} \cdot \underset{1}{\vec{z}_0} \wedge \vec{y}_1 = -\dot{\psi} \cdot \vec{x}_1$$

$$\left.\frac{d\vec{z}_2}{dt}\right|_0 = \vec{\Omega}_{2/0} \wedge \vec{z}_2 = \left(\dot{\theta} \cdot \underset{2}{\vec{x}_1} + \dot{\psi} \cdot \underset{1}{\vec{z}_0}\right) \wedge \vec{z}_2 = -\dot{\theta} \cdot \vec{y}_2 + \dot{\psi} sin\theta \cdot \underset{2}{\vec{x}_1}$$

$$\left.\frac{d\vec{x}_3}{dt}\right|_0 = \vec{\Omega}_{3/0} \wedge \vec{x}_3 = \left(\dot{\varphi} \cdot \underset{3}{\vec{z}_2} + \dot{\theta} \cdot \underset{2}{\vec{x}_1} + \dot{\psi} \cdot \underset{1}{\vec{z}_0}\right) \wedge \vec{x}_3 = \dot{\varphi} \cdot \vec{y}_3 + \dot{\theta} sin\varphi \cdot \underset{3}{\vec{z}_2} + \dot{\psi} \cdot \underset{1}{\vec{z}_0} \wedge \vec{x}_3$$

$$\underset{1}{\vec{z}_0} \wedge \vec{x}_3 = \left(sin\theta \cdot \vec{y}_2 + cos\theta \cdot \underset{3}{\vec{z}_2}\right) \wedge \vec{x}_3 = -sin\theta cos\varphi \cdot \underset{3}{\vec{z}_2} + cos\theta \cdot \vec{y}_3$$

Ou :

$$\underset{1}{\vec{z}_0} \wedge \vec{x}_3 = \underset{1}{\vec{z}_0} \wedge \left(cos\varphi \cdot \underset{2}{\vec{x}_1} + sin\varphi \cdot \vec{y}_2\right) = cos\varphi \cdot \vec{y}_1 - sin\varphi cos\theta \cdot \underset{2}{\vec{x}_1}$$

D'où

$$\boxed{\vec{V}_{D,3/0} = -\dot{\psi}(a + bsin\theta) \cdot \underset{2}{\vec{x}_1} + \dot{\lambda} \cdot \vec{z}_1 + b\dot{\theta} \cdot \vec{y}_2 + c\left(\dot{\theta} sin\varphi - \dot{\psi} sin\theta cos\varphi\right) \cdot \underset{3}{\vec{z}_2} + c \cdot \left(\dot{\varphi} + \dot{\psi} cos\theta\right) \cdot \vec{y}_3}$$

Ou :

$$\boxed{\vec{V}_{D,3/0} = -\dot{\psi}(a + bsin\theta + csin\varphi cos\theta) \cdot \underset{2}{\vec{x}_1} + c\dot{\psi} cos\varphi \cdot \vec{y}_1 + \dot{\lambda} \cdot \vec{z}_1 + b\dot{\theta} \cdot \vec{y}_2 + c\dot{\theta} sin\varphi \cdot \underset{3}{\vec{z}_2} + c\dot{\varphi} \cdot \vec{y}_3}$$

$\vec{\Gamma}_{D,3/0} = \left.\dfrac{d\vec{V}_{D,3/0}}{dt}\right]_0$ avec, par hypothèse : $\dot{\psi}$ constant

Or :

$$\vec{V}_{D,3/0} = -\dot{\psi}(a + b\sin\theta)\cdot\underset{2}{\vec{x}_1} + \dot{\lambda}\cdot\vec{z}_1 + b\dot{\theta}\cdot\vec{y}_2 + c\left(\dot{\theta}\sin\varphi - \dot{\psi}\sin\theta\cos\varphi\right)\cdot\underset{3}{\vec{z}_2} + c\cdot\left(\dot{\varphi} + \dot{\psi}\cos\theta\right)\cdot\vec{y}_3$$
$$= A\cdot\underset{2}{\vec{x}_1} + B\cdot\vec{z}_1 + C\cdot\vec{y}_2 + D\cdot\underset{3}{\vec{z}_2} + E\cdot\vec{y}_3$$

Avec :

$\left.\dfrac{d\vec{x}_1}{dt}\right]_0 = \dot{\psi}\cdot\vec{y}_1$ $\left.\dfrac{d\vec{z}_1}{dt}\right]_0 = \vec{0}$ $\left.\dfrac{d\vec{y}_2}{dt}\right]_0 = \vec{\Omega}_{2/0}\wedge\vec{y}_2 = \left(\dot{\theta}\cdot\underset{2}{\vec{x}_1} + \dot{\psi}\cdot\underset{1}{\vec{z}_0}\right)\wedge\vec{y}_2 = \dot{\theta}\cdot\vec{z}_2 - \dot{\psi}\cos\theta\cdot\vec{x}_1$

$$\left.\frac{d\vec{z}_2}{dt}\right]_0 = \vec{\Omega}_{2/0}\wedge\vec{z}_2 = \left(\dot{\theta}\cdot\underset{2}{\vec{x}_1} + \dot{\psi}\cdot\underset{1}{\vec{z}_0}\right)\wedge\vec{z}_2 = -\dot{\theta}\cdot\vec{y}_2 + \dot{\psi}\sin\theta\cdot\underset{2}{\vec{x}_1}$$

$$\left.\frac{d\vec{y}_3}{dt}\right]_0 = \vec{\Omega}_{3/0}\wedge\vec{y}_3 = \left(\dot{\varphi}\cdot\underset{3}{\vec{z}_2} + \dot{\theta}\cdot\underset{2}{\vec{x}_1} + \dot{\psi}\cdot\underset{1}{\vec{z}_0}\right)\wedge\vec{y}_3 = -\dot{\varphi}\cdot\vec{x}_3 + \dot{\theta}\cos\varphi\cdot\underset{3}{\vec{z}_2} + \dot{\psi}\cdot\underset{1}{\vec{z}_0}\wedge\vec{y}_3$$

$$\underset{1}{\vec{z}_0}\wedge\vec{y}_3 = \left(\sin\theta\cdot\vec{y}_2 + \cos\theta\cdot\underset{3}{\vec{z}_2}\right)\wedge\vec{y}_3 = \sin\theta\sin\varphi\cdot\underset{3}{\vec{z}_2} - \cos\theta\cdot\vec{x}_3$$

Ou :

$$\underset{1}{\vec{z}_0}\wedge\vec{y}_3 = \underset{1}{\vec{z}_0}\wedge\left(-\sin\varphi\cdot\underset{2}{\vec{x}_1} + \cos\varphi\cdot\vec{y}_2\right) = -\sin\varphi\cdot\vec{y}_1 - \cos\varphi\cos\theta\cdot\underset{2}{\vec{x}_1}$$

$\dfrac{dA}{dt} = -b\dot{\psi}\dot{\theta}\cos\theta$ $\dfrac{dB}{dt} = \ddot{\lambda}$ $\dfrac{dC}{dt} = b\ddot{\theta}$

$\dfrac{dD}{dt} = c\left[\ddot{\theta}\sin\varphi + \dot{\theta}\dot{\varphi}\cos\varphi - \dot{\psi}\left(\dot{\theta}\cos\theta\cos\varphi - \dot{\varphi}\sin\theta\sin\varphi\right)\right]$ $\dfrac{dE}{dt} = c\left(\ddot{\varphi} - \dot{\psi}\dot{\theta}\sin\theta\right)$

	1/0	**2/1**	**3/2**
Liaison normalisée	**Pivot glissant d'axe $(O,\vec{z}_0)$**	**Pivot d'axe $(B,\vec{x}_1)$**	**Pivot d'axe $(C,\vec{z}_2)$**
Torseur cinématique	$\{V_{1/0}\} = \left\{\begin{matrix}\dot{\psi}\cdot\underset{1}{\vec{z}_0} \\ \dot{\lambda}\cdot\underset{1}{\vec{z}_0}\end{matrix}\right\}_{OouA}$	$\{V_{2/1}\} = \left\{\begin{matrix}\dot{\theta}\cdot\underset{2}{\vec{x}_1} \\ \vec{0}\end{matrix}\right\}_{B}$	$\{V_{3/2}\} = \left\{\begin{matrix}\dot{\varphi}\cdot\underset{3}{\vec{z}_2} \\ \vec{0}\end{matrix}\right\}_{C}$

Expression de $\overrightarrow{V_{D,3/0}}$ par <u>composition des mouvements</u> + <u>champ des vecteurs vitesse</u>

$$\overrightarrow{V_{D,3/0}} = \overrightarrow{V_{D,3/2}} + \overrightarrow{V_{D,2/1}} + \overrightarrow{V_{D,1/0}}$$

$$\overrightarrow{V_{D,3/2}} = \overrightarrow{DC}\wedge\vec{\Omega}_{3/2} = -c.\vec{x}_3\wedge\dot{\varphi}\cdot\underset{3}{\vec{z}_2} = c\dot{\varphi}\cdot\vec{y}_3$$

$$\overrightarrow{V_{D,2/1}} = \overrightarrow{DB}\wedge\vec{\Omega}_{2/1} = (-c.\vec{x}_3 + b.\vec{z}_2)\wedge\dot{\theta}\cdot\underset{2}{\vec{x}_1} = \dot{\theta}\left(c\sin\varphi\cdot\underset{3}{\vec{z}_2} + b.\vec{y}_2\right)$$

$$\overrightarrow{V_{D,1/0}} = \overrightarrow{V_{A,1/0}} + \overrightarrow{DA}\wedge\vec{\Omega}_{1/0} = \dot{\lambda}\cdot\underset{1}{\vec{z}_0} + \left(-c.\vec{x}_3 + b.\vec{z}_2 - a.\vec{y}_1\right)\wedge\dot{\psi}\cdot\underset{1}{\vec{z}_0}$$
$$= \dot{\lambda}\cdot\underset{1}{\vec{z}_0} - c.\vec{x}_3\wedge\dot{\psi}\cdot\underset{1}{\vec{z}_0} + \dot{\psi}\cdot\left(-b\sin\theta\cdot\underset{2}{\vec{x}_1} - a\cdot\underset{2}{\vec{x}_1}\right)$$
$$= \dot{\lambda}\cdot\underset{1}{\vec{z}_0} + c\dot{\psi}\left(-\sin\theta\cos\varphi\cdot\underset{3}{\vec{z}_2} + \cos\theta\cdot\vec{y}_3\right) - \dot{\psi}\cdot(b\sin\theta + a)\cdot\underset{2}{\vec{x}_1}$$

Ou :

$$= \dot{\lambda}\cdot\underset{1}{\vec{z}_0} + c\dot{\psi}\left(\cos\varphi\cdot\vec{y}_1 - \sin\varphi\cos\theta\cdot\underset{2}{\vec{x}_1}\right) - \dot{\psi}\cdot(b\sin\theta + a)\cdot\underset{2}{\vec{x}_1}$$

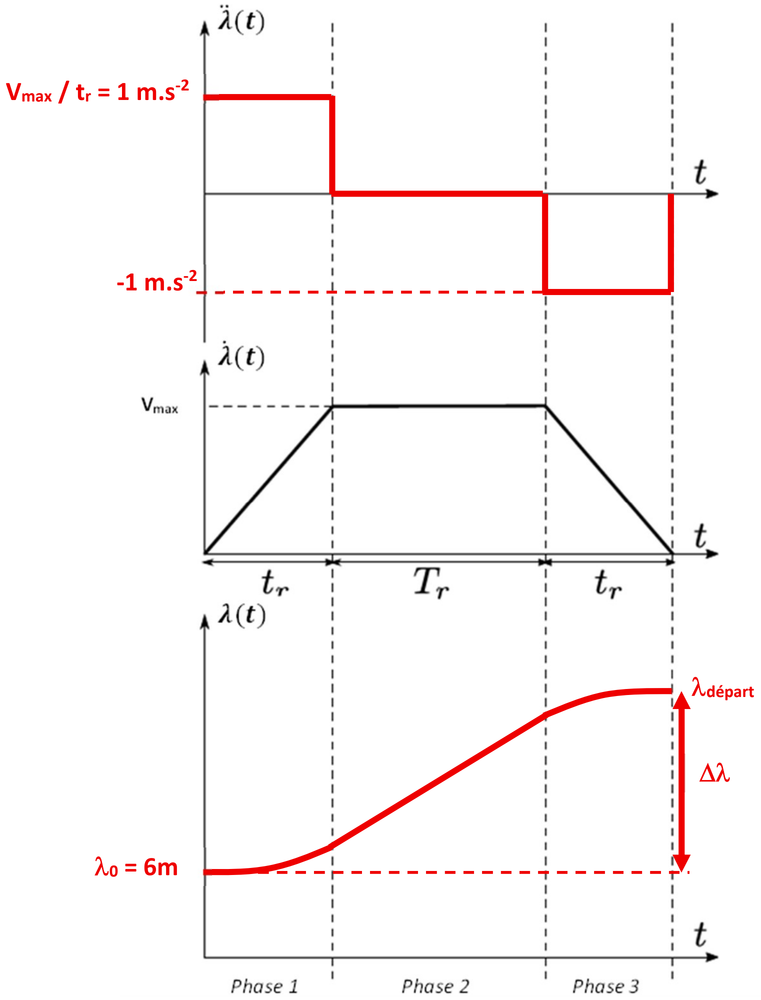
On retrouve alors l'expression de $\overrightarrow{V_{D,3/0}}$ déterminée en Q3.

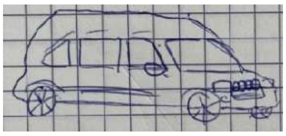
$$\{V_{3/0}\} = \left\{\begin{matrix}\dot{\psi}\cdot\underset{1}{\vec{z}_0} + \dot{\theta}\cdot\underset{2}{\vec{x}_1} + \dot{\varphi}\cdot\underset{3}{\vec{z}_2} \\ \dot{\psi}(a + b\sin\theta)\cdot\underset{2}{\vec{x}_1} + \dot{\lambda}\cdot\vec{z}_1 + b\dot{\theta}\cdot\vec{y}_2 + c\left(\dot{\theta}\sin\varphi - \dot{\psi}\sin\theta\cos\varphi\right)\cdot\underset{3}{\vec{z}_2} + c\cdot\left(\dot{\varphi} + \dot{\psi}\cos\theta\right)\cdot\vec{y}_3\end{matrix}\right\}_D$$

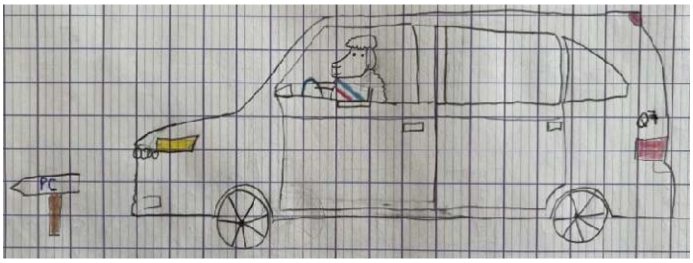
$\Delta\lambda = \int_{t=0}^{t=T_r+2t_r} \dot{\lambda}(t).dt$ soit l'aire sous le profil de vitesse en trapèze.

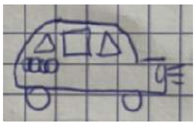
Donc $\Delta\lambda = V_{max}(t_r + T_r)$ = 6 m.

Donc $\lambda_{départ} = \lambda_0 + \Delta\lambda$ = 12 m.






PC
Q7

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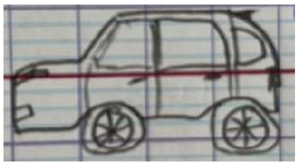

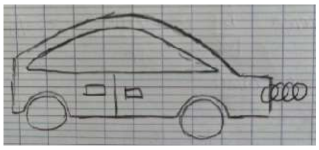

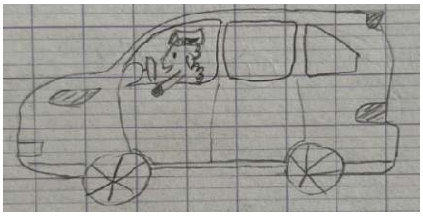

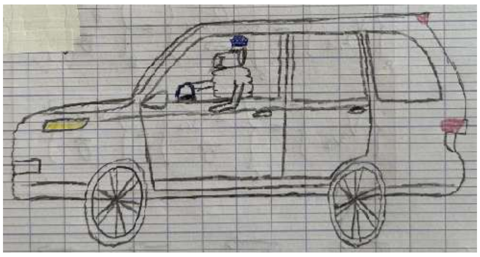

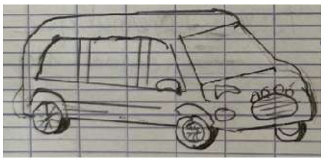

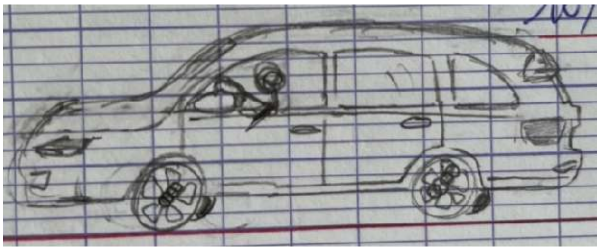

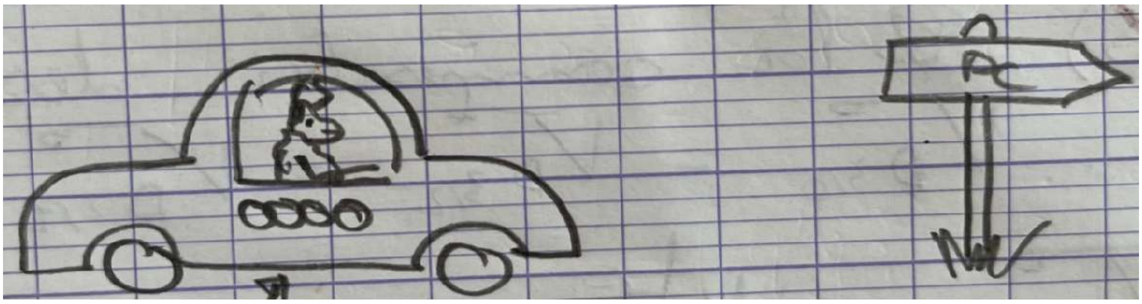
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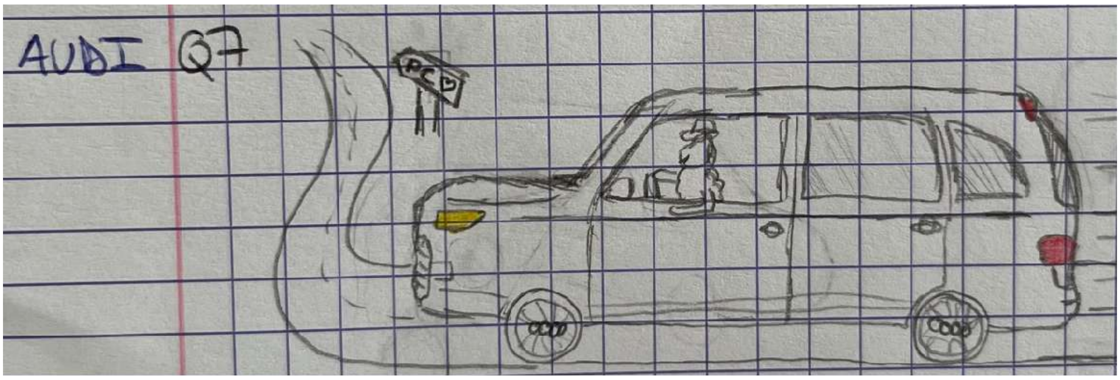
AUDI Q7
PC