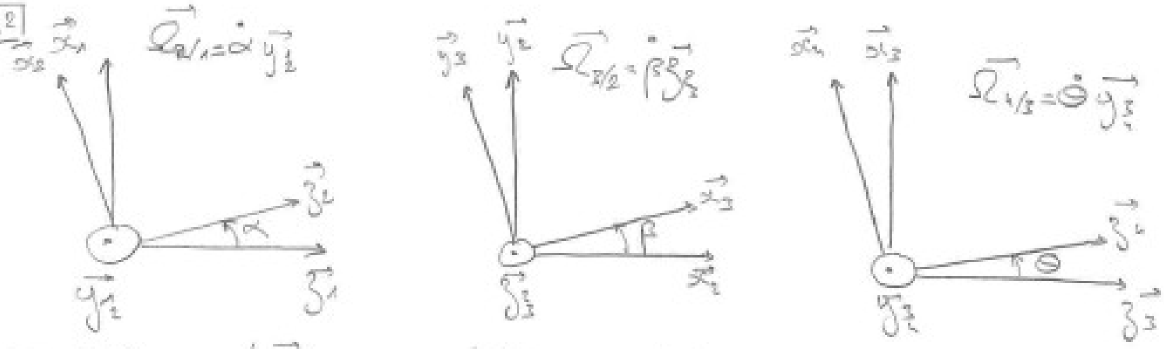
Manège StarLight *Corrigé*

Q1

Q2



Q3 $\vec{V}_{B,2/1} = \left.\frac{d\vec{AB}}{dt}\right]_{R_1} = a\left.\frac{d\vec{y_2}}{dt}\right]_{R_1} + b\left.\frac{d\vec{z_2}}{dt}\right]_{R_1}$

$= b\dot\alpha\,\vec{x_2}$

Q4 $\vec{V}_{C,3/1} = \left.\frac{d\vec{AC}}{dt}\right]_{R_1} = \left.\frac{d\vec{AB}}{dt}\right]_{R_1} + \left.\frac{d\vec{BC}}{dt}\right]_{R_1}$

$= b\dot\alpha\,\vec{x_2} - c\left.\frac{d\vec{y_3}}{dt}\right]_{R_1} - d\left.\frac{d\vec{z_2}}{dt}\right]_{R_1}$

$= b\dot\alpha\,\vec{x_2} - c\,\vec{\Omega}_{3/1}\wedge\vec{y_3} - d\dot\alpha\,\vec{x_2}$

$= \text{———} - c\left(\dot\beta\,\vec{z_2} + \dot\alpha\,\vec{y_1}\right)\wedge\vec{y_3} + \text{———}$

$= \dot\alpha(b-d)\,\vec{x_2} + c\dot\beta\,\vec{x_3} - c\dot\alpha\sin\beta\,\vec{z_3}$

Q5 $\vec{V}_{P,4/1} = \left.\frac{d\vec{AP}}{dt}\right]_{R_1} = \left.\frac{d\vec{AC}}{dt}\right]_{R_1} + \left.\frac{d\vec{CP}}{dt}\right]_{R_1}$

$= \vec{V}_{C,3/1} + e\left.\frac{d\vec{y_4}}{dt}\right]_{R_1} + f\left.\frac{d\vec{z_4}}{dt}\right]_{R_1}$

avec ① $\left.\frac{d\vec{y_4}}{dt}\right]_{R_1} = \vec{\Omega}_{4/1}\wedge\vec{y_4} = \left(\dot\theta\,\vec{y_3} + \dot\beta\,\vec{z_2} + \dot\alpha\,\vec{y_1}\right)\wedge\vec{y_3}$

$= -\dot\beta\,\vec{x_3} + \dot\alpha\sin\beta\,\vec{z_3}$ $(\times e)$

② $\left.\frac{d\vec{z_4}}{dt}\right]_{R_1} = \vec{\Omega}_{4/1}\wedge\vec{z_4} = \left(\dot\theta\,\vec{y_3} + \dot\beta\,\vec{z_2} + \dot\alpha\,\vec{y_1}\right)\wedge\vec{z_4}$

$= \dot\theta\,\vec{x_4} + \dot\beta\sin\theta\,\vec{y_3} + \dot\alpha\,\underbrace{\vec{y_2}\wedge\vec{z_4}}$

$= \vec{y_2}\wedge\left(\sin\theta\,\vec{x_3} + \cos\theta\,\vec{z_3}\right)$

$= -\sin\theta\cos\beta\,\vec{z_2} + \cos\theta\,\vec{x_2}$ $(\times f)$

Donc

$\vec{V}_{P,4/1} = \left(b\dot\alpha - d\dot\alpha + f\dot\alpha\cos\theta\right)\vec{x_2} + \left(-c\dot\alpha\sin\beta + e\dot\alpha\sin\beta - f\dot\alpha\sin\theta\cos\beta\right)\vec{z_2} + \left(c\dot\beta - e\dot\beta\right)\vec{x_3}$
$+ f\dot\beta\sin\theta\,\vec{y_3} + f\dot\theta\,\vec{x_4}$

$\boxed{Q6}$ $\vec{\Gamma}_{P,4/1} = \left.\dfrac{d\vec{V}_{P,4/1}}{dt}\right|_{R_1}$ (on suppose $\dot{\alpha}$, $\dot{\beta}$ et $\dot{\theta}$ constants)

$$= -f\dot{\alpha}\dot{\theta}\sin\theta\,\vec{x_2} + \left[\dot{\alpha}\dot{\beta}\cos\beta(e-c) - f\dot{\alpha}\left(\dot{\theta}\cos\theta\cos\beta - \dot{\beta}\sin\theta\sin\beta\right)\right]\vec{z_2}$$

$$+ f\dot{\beta}\dot{\theta}\cos\theta\,\vec{y_3}$$

$$+ \dot{\alpha}(b-d+f\cos\theta)\left.\frac{d\vec{x_2}}{dt}\right|_{R_1} + \dot{\alpha}(-c\sin\beta + e\sin\beta - f\sin\theta\cos\beta)\left.\frac{d\vec{z_2}}{dt}\right|_{R_1}$$

$$+ \dot{\beta}(c-e)\left.\frac{d\vec{x_3}}{dt}\right|_{R_1} + f\dot{\beta}\sin\theta\left.\frac{d\vec{y_3}}{dt}\right|_{R_1} + f\dot{\theta}\left.\frac{d\vec{x_4}}{dt}\right|_{R_1}$$

avec $\left.\dfrac{d\vec{x_2}}{dt}\right|_{R_1} = -\dot{\alpha}\,\vec{z_2}$

$\left.\dfrac{d\vec{z_2}}{dt}\right|_{R_1} = \dot{\alpha}\,\vec{x_2}$

$\left.\dfrac{d\vec{x_3}}{dt}\right|_{R_1} = \vec{\Omega}_{3/1}\wedge\vec{x_3} = \left(\dot{\beta}\,\vec{z_3} + \dot{\alpha}\,\vec{y_1}\right)\wedge\vec{x_3} = \dot{\beta}\,\vec{y_3} - \dot{\alpha}\cos\beta\,\vec{z_3}$

$\left.\dfrac{d\vec{y_3}}{dt}\right|_{R_1} = \vec{\Omega}_{3/1}\wedge\vec{y_3} = \left(\dot{\beta}\,\vec{z_3} + \dot{\alpha}\,\vec{y_1}\right)\wedge\vec{y_3} = -\dot{\beta}\,\vec{x_3} + \dot{\alpha}\sin\beta\,\vec{z_3}$

$\left.\dfrac{d\vec{x_4}}{dt}\right|_{R_1} = \vec{\Omega}_{4/1}\wedge\vec{x_4} = \left(\dot{\theta}\,\vec{y_3} + \dot{\beta}\,\vec{z_3} + \dot{\alpha}\,\vec{y_1}\right)\wedge\vec{x_4} = -\dot{\theta}\,\vec{z_4} + \dot{\beta}\cos\theta\,\vec{y_3} + \dot{\alpha}\left(\sin\beta\sin\theta\,\vec{y_3} - \cos\beta\,\vec{z_4}\right)$

D'où

$$\vec{\Gamma}_{P,4/1} = \left[-f\dot{\alpha}\dot{\theta}\sin\theta + \dot{\alpha}^2(-c\sin\beta + e\sin\beta - f\sin\theta\cos\beta)\right]\vec{x_2}$$

$$+ \Big[\dot{\alpha}\dot{\beta}\cos\beta(e-c) - f\dot{\alpha}\left(\dot{\theta}\cos\theta\cos\beta - \dot{\beta}\sin\theta\sin\beta\right) - \dot{\alpha}^2(b-d+f\cos\theta)$$

$$- \dot{\alpha}\dot{\beta}(c-e)\cos\beta + f\dot{\beta}\dot{\alpha}\sin\theta\sin\beta\Big]\vec{z_2}$$

$$- f\dot{\beta}^2\sin\theta\,\vec{x_3} + \left[f\dot{\beta}\dot{\theta}\cos\theta + \dot{\beta}^2(c-e) + f\dot{\theta}\left(\dot{\beta}\cos\theta + \dot{\alpha}\sin\beta\sin\theta\right)\right]\vec{y_3}$$

$$+ f\dot{\theta}\left(-\dot{\theta} - \dot{\alpha}\cos\beta\right)\vec{z_4}$$

$\boxed{Q8}$

Composition des mouvements :

$$\overrightarrow{V_{P,4/1}} = \overrightarrow{V_{P,4/3}} + \overrightarrow{V_{P,3/2}} + \overrightarrow{V_{P,2/1}}$$

avec

③ $\overrightarrow{V_{P,4/3}} = \overrightarrow{V_{C,4/3}} + \overrightarrow{PC} \wedge \overrightarrow{\Omega_{4/3}}$

$= (-e\overrightarrow{y_4} - f\overrightarrow{z_4}) \wedge \dot{\theta}\, \overrightarrow{y_{\substack{3\\4}}}$

$= +f\dot{\theta}\, \overrightarrow{x_4}$

② $\overrightarrow{V_{P,3/2}} = \overrightarrow{V_{B,3/2}} + \overrightarrow{PB} \wedge \overrightarrow{\Omega_{3/2}}$

$= (-e\overrightarrow{y_{\substack{3\\4}}} - f\overrightarrow{z_4} + c\overrightarrow{y_3} + d\overrightarrow{z_3}) \wedge \dot{\beta}\, \overrightarrow{z_{\substack{2\\3}}}$

$= \dot{\beta}\,(-e\overrightarrow{x_3} + f\sin\theta\, \overrightarrow{y_{\substack{3\\4}}} + c\overrightarrow{x_3})$

① $\overrightarrow{V_{P,2/1}} = \overrightarrow{V_{A,2/1}} + \overrightarrow{PA} \wedge \overrightarrow{\Omega_{2/1}}$

$= (-e\overrightarrow{y_{\substack{3\\4}}} - f\overrightarrow{z_4} + c\overrightarrow{y_3} + d\overrightarrow{z_3} - a\overrightarrow{y_2} - b\overrightarrow{z_2}) \wedge \dot{\alpha}\, \overrightarrow{y_{\substack{1\\2}}}$

$= \dot{\alpha}\left[e\sin\beta\, \overrightarrow{z_{\substack{2\\3}}} - f(\sin\theta\cos\beta\, \overrightarrow{z_{\substack{2\\3}}} - \cos\theta\, \overrightarrow{x_2}) - c\sin\beta\, \overrightarrow{z_{\substack{2\\3}}} - d\overrightarrow{x_2} + b\overrightarrow{x_2}\right]$

$\overrightarrow{z_4} \wedge \overrightarrow{y_2} = (\sin\theta\, \overrightarrow{x_3} + \cos\theta\, \overrightarrow{z_{\substack{3\\4}}}) \wedge \overrightarrow{y_2}$

$= \sin\theta\cos\beta\, \overrightarrow{z_{\substack{2\\3}}} - \cos\theta\, \overrightarrow{x_2}$

$\boxed{Q7}$

$$\{\mathcal{V}_{2/1}\} = \begin{Bmatrix} \dot{\alpha}\,\vec{y_2} \\ \vec{0} \end{Bmatrix}_A \qquad \{\mathcal{V}_{3/2}\} = \begin{Bmatrix} \dot{\beta}\,\vec{z_3} \\ \vec{0} \end{Bmatrix}_B \qquad \{\mathcal{V}_{4/3}\} = \begin{Bmatrix} \dot{\theta}\,\vec{y_3} \\ \vec{0} \end{Bmatrix}_C$$

$\boxed{Q9}$

$$\{\mathcal{V}_{4/1}\} = \begin{Bmatrix} \dot{\alpha}\,\vec{y_2} + \dot{\beta}\,\vec{z_3} + \dot{\theta}\,\vec{y_3} \\ \overrightarrow{V_{P,4/1}} \end{Bmatrix}_P$$

obtenu en $\boxed{Q5}$ ou $\boxed{Q8}$

$\boxed{Q10}$

$$\|\overrightarrow{V_{P,4/1}}\|^2 = \overrightarrow{V_{P,4/1}} \cdot \overrightarrow{V_{P,4/1}}$$

$$= \left(V_{x2}\,\vec{x_2} + V_{z2}\,\vec{z_2} + V_{x3}\,\vec{x_3} + V_{y3}\,\vec{y_3} + V_{x4}\,\vec{x_4}\right)$$

$$\cdot \left(V_{x2}\,\vec{x_2} + V_{z2}\,\vec{z_2} + V_{x3}\,\vec{x_3} + V_{y3}\,\vec{y_3} + V_{x4}\,\vec{x_4}\right)$$

$$= V_{x2}^2 + V_{z2}^2 + V_{x3}^2 + V_{y3}^2 + V_{x4}^2 + 2\Big(V_{x2}V_{z2}\,\cancel{\vec{x_2}\cdot\vec{z_2}}_{0} + V_{x2}V_{x3}\,\vec{x_2}\cdot\vec{x_3}$$

$$+ V_{x2}V_{y3}\,\vec{x_2}\cdot\vec{y_3} + V_{x2}V_{x4}\,\vec{x_2}\cdot\vec{x_4} + V_{z2}V_{x3}\,\cancel{\vec{z_2}\cdot\vec{x_3}}_{0} + V_{z2}V_{y3}\,\cancel{\vec{z_2}\cdot\vec{y_3}}_{0}$$

$$+ V_{z2}V_{x4}\,\vec{z_2}\cdot\vec{x_4} + V_{x3}V_{y3}\,\cancel{\vec{x_3}\cdot\vec{y_3}}_{0} + V_{x3}V_{x4}\,\vec{x_3}\cdot\vec{x_4} + V_{y3}V_{x4}\,\cancel{\vec{y_3}\cdot\vec{x_4}}_{0}\Big)$$

avec $\vec{x_2}\cdot\vec{x_3} = \cos\beta$

$\vec{x_2}\cdot\vec{y_3} = -\sin\beta$

$\vec{x_2}\cdot\vec{x_4} = \cos\beta\cos\theta$

$\vec{z_2}\cdot\vec{x_4} = -\sin\theta$

$\vec{x_4}\cdot\vec{x_4} = \cos\theta$

D'où

$$\|V_{P,4/1}\|^2 = V_{x2}^2 + V_{z2}^2 + V_{x3}^2 + V_{y3}^2 + V_{x4}^2$$

$$+ 2\left(V_{x2}V_{x3}\cos\beta - V_{x2}V_{y3}\sin\beta + V_{x2}V_{x4}\cos\beta\cos\theta - V_{z2}V_{x4}\sin\theta\right.$$

$$\left. + V_{x3}V_{x4}\cos\theta\right)$$

$\boxed{Q11}$ Le passager extrême est celui qui subit l'accélération $\|\vec{\Gamma}_{P,4/0}\|$ la plus importante

En effet, d'après $\boxed{Q6}$ on constate que cette accélération dépend de la position du passager sur la nacelle 4 (coordonnées e et f)

$\boxed{Q12}$ On lit sur le graphe $\|\vec{\Gamma}_{P,4/0}\|_{max} \simeq 18,2 \text{ m.s}^{-2}$

$$\simeq \frac{18,2}{9,81} g \simeq 1,9 g < 2,5 g$$

$\Rightarrow \boxed{\text{CDC OK}}$